Garyounis University -- Faculty of Engineering -- Electrical department

Linear System (EE 311) - Spring 2010 - Solution of Final Exam

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Q1

(a)

$$k3^k, k \ge 0 \leftrightarrow -z\frac{d}{dz}(1-3z^{-1})^{-1}$$

$$Y(z) = \frac{3z^{-1}}{(1-3z^{-1})^2}$$

(b)

$$Y(s) = \frac{1}{(s+3)^2} \qquad \mathrm{Re}(s) > -3$$

Q2

$$Y(z) = U(z)H^1(z)H^2(z) = \frac{z}{z-1}\frac{z}{z+0.8}\frac{z}{z-0.8}$$

$$Y(z) = 0.4\left[\frac{2.7z}{z-1} + \frac{0.222z}{z+0.8} - \frac{2z}{z-0.8}\right]$$

$$h_k = 0.4\left[2.7 + 0.22(-0.8)^k - 2(0.8)^k\right]u_k = \left[1.1 + 0.0889(-0.8)^k - 0.8(0.8)^k\right]u_k$$

$$y_k = u_k * h_k^1 * h_k^2 = \{0.4, 0.4, 0.656, 0.656, 0.8198, 0.8198, \ldots\}$$

Q3

The state equations are

$$x_1(k+1) = x_1(k) + Kx_2(k) + u(k)$$
$$x_2(k+1) = x_2(k) + \frac{1}{6}x_2(k)$$

$$A = \begin{bmatrix} 1 & K \\ 1 & \frac{1}{6} \end{bmatrix}$$

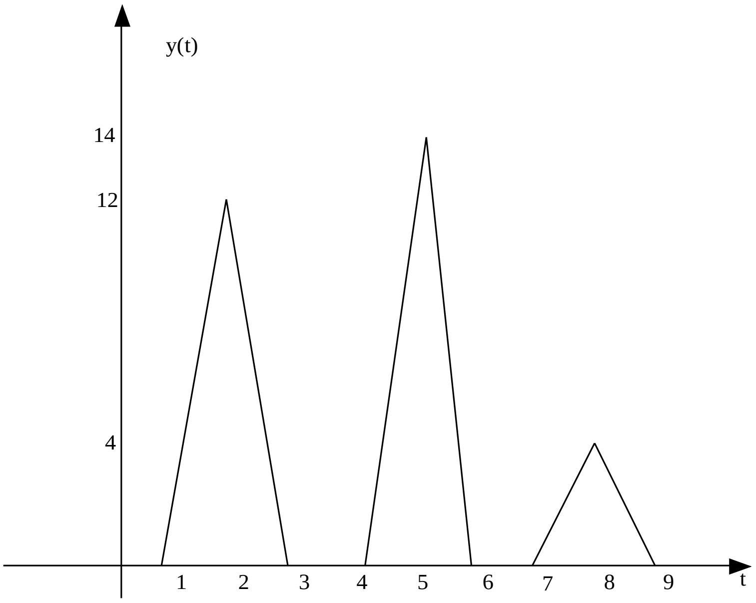
$$g(\lambda) = \lambda^2 - \frac{7}{6}\lambda + \frac{1}{6} - K = 0$$

$$\lambda_{1,2} = \frac{7}{12} \pm \sqrt{\frac{25}{144} + K}$$

The system is stable for all values of K in the range

$$-\frac{5}{6} < K < 0$$

Q4) $$y_k = u_k * h_k$$



Q5)

(a) $$Q(s) = s^2 + 2s - 24 + g = 0$$

$$s_{1,2} = -1 \pm \sqrt{25 - g}$$

Thus the system is stable for

$$g > 24$$

(b) For g =25, and Re(s) > 4

$$H(s) = \frac{s^2 + 4s}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + 1$$

$$H(s) = \frac{2}{s+1} + \frac{-3}{(s+1)^2} + 1$$

$$h(t) = \delta(t) + (2e^{-t} - 3te^{-t})u(t)$$

Both poles lie to the left of convergence region. Therefore the signal is causal

Q6)
a)

$$x_1' = -5x_1 - 5x_2 + 5u_1$$

$$x_2' = \tfrac{1}{c}x_1 + \tfrac{1}{2c}x_2 - \tfrac{1}{c}u_2$$

$$y_1 = x_1 \qquad y_3 = -5u_2$$

$$y_2 = x_2 \qquad y_4 = \frac{x_2}{2} - u_2$$

$$A = \begin{bmatrix} -5 & -5 \\ \frac{1}{c} & \frac{1}{2c} \end{bmatrix} \qquad B = \begin{bmatrix} -5 & 0 \\ 0 & \frac{-1}{c} \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -5 \\ 0 & -1 \end{bmatrix}$$

b)

if c=0.2F $\quad A = \begin{bmatrix} -5 & -5 \\ 5 & 2.5 \end{bmatrix}$

$\lambda_{1,2}$= -1.2500 ± 3.3072i ➔ the system is stable

if c=1mF $\quad A = \begin{bmatrix} -5 & -5 \\ 1000 & 500 \end{bmatrix}$

λ_1= 5.1031
λ_2= 489.8969

➔ the system is unstable